Math 260: Linear Algebra
Chapter 5 & 6 Section 1 – Vector Spaces
Homework

1. Prove vector space properties A1 through A5 are true for the vector space $\mathbb{R}^3$ with the usual rule for vector addition and scalar multiplication.

2. Prove vector space properties S1 through S5 are true for the vector space P_1 with the usual rule for vector addition and scalar multiplication.

3. (book section 6.1 #1ac)

Let V denote the set of ordered triples (x, y, z) and define addition in V as in $\mathbb{R}^3$. For each of the following definitions of scalar multiplication, decide whether V is a vector space.

a. $a(x, y, z) = (ax, y, az)$

c. $a(x, y, z) = (0, 0, 0)$

4. (book section 6.1 #2abl)

Are the following sets vector spaces with the indicated operations? If not, why not?

a. The set V of nonnegative real numbers; ordinary addition and scalar multiplication.

b. The set V of all polynomials of degree ≥ 3, together with 0; operations of $\mathbf{P}$.

l. The set V of all functions $f : \mathbb{R} \to \mathbb{R}$ with pointwise addition, but scalar multiplication defined by $(af)(x) = f(ax)$.

5. (book section 6.1 #3)

Let V be the set of positive real numbers with vector addition being ordinary multiplication, and scalar multiplication being $a \cdot v = v^a$. Show that V is a vector space.

6. (book section 6.1 #4)

If V is the set of ordered pairs $(x,\ y)$ of real numbers, show that it is a vector space with addition $(x,\ y)+(x_1,\ y_1)=(x+x_1,\ y+y_1+1)$ and scalar multiplication $a(x,\ y)=(ax,\ ay+a-1)$.